LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2018**

16/17/18UMT1MC02 – ANALYTICAL GEOMETRY OF 2D, TRIG. MATRICES

Date: 31-10-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL questions (10 X 2 = 20)

1. Write the expansion for $\cos n\theta$.

2. If $x = \cos\theta + i\sin\theta$ find $x^n + \frac{1}{x^n}$.

3. Prove that $\cosh^2 x - \sinh^2 x = 1$.

4. Find the value of log(4 + 3i).

5. State Cayley Hamilton theorem.

6. Define the eigen values of the matrix.

7. Find the pole of the line Ax + By + C = 0 with respect to the parabola $y^2 = 4ax$.

8. Define the conjugate diameters of the ellipse.

9. Write the standard form of the equation to the hyperbola and its asymptotes.

10. Find the centre of the hyperbola $9x^2 - 16y^2 + 72x - 32y - 16 = 0$.

PART – B

Answer any FIVE questions (5 X 8 = 40)

11. Expand $\cos 6\theta$ in terms of $\sin\theta$

12. Evaluate $\lim_{x\to 0}\frac{\sin 2x - 2\sin x}{x^3}$

13. Separate into real and imaginary parts $\tan^{-1}(x+iy)$

14. Express $\cosh^6\theta$ in terms of hyperbolic cosines of multiples of θ.

15. Show that the matrix $\begin{bmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 2\end{bmatrix}$ satisfies Cayley Hamilton theorem.

16. Find the characteristic roots of the matrix $\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$.

17. Prove that in an ellipse, the tangents at the extremities of a diameter are parallel to the chords bisected by the diameter.

18. Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$

PART– C
Answer Any TWO Questions (2 X 20 = 40)

19. a. Find $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$.

b. Expand $\sin^3\theta\cos^4\theta$ in terms of sines of multiples of θ. **(8 + 12)**

20. a. If $\cosh u=\sec\theta$, show that $u=\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$.

b. If $\tan(x+iy)=u+iv$, prove that $\frac{u}{v}=\frac{\sin 2x}{\sinh 2y}$. **(8 + 12)**

21. Diagonalize the matrix $A=\begin{bmatrix}2 & -2 & 3\\ 1 & 1 & 1\\ 1 & 3 & -1\end{bmatrix}$

22. a. Show that the locus of the intersection of the tangents to $y^2=4ax$ which intercepts a constant length d on the directrix is $(y^2-4ax)(x+a)^2=d^2x^2$.

b. Find the locus of the foot of the perpendiculars drawn from the pole to the tangents to the circle $=a(1+cos\theta)$. **(10 + 10)**

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